INTRODUCTION

We have learnt about number system, rational and irrational numbers, number line, real numbers and operations on the number line. Now we will discuss in detail about Euclid's division algorithm and fundamental theorem of arithmetic.

Euclid's division algorithm, says that any positive integer a can be divided by another positive integer b in such a way that it leaves a remainder r that is smaller than b. We use this result to obtain the H.C.F. of two numbers.

1. EUCLID'S DIVISION LEMMA

Dividend = divisor × quotient + remainder.

Given two positive integers a and b. There exist unique integers q and r satisfying

$a = bq + r$ where $0 \le r < b$

where a is dividend, b is divisor, q is quotient and r is remainder.

The most important application to this algorithm is to find H.C.F. of two given positive integers.

If we divide 119 by 8, we get 14 as quotient and 7 as remainder.

∴ 119 = (8 × 14) + 7

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     14
8 ) 119
     8
     39
     32
      7
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2. EUCLID'S DIVISION ALGORITHM

An algo is a process of solving particular problems. According to this algo, to obtain the H.C.F. of two positive integers, say c and d, with $c > d$, follow the steps below:

Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.

Step 2: If $r = 0$, d is the H.C.F. of c and d. If $r \neq 0$, apply the division lemma to d and r.

Step 3: Write $d = er + r_1$ where $0 < r_1 < r$.

Step 4: Continue the process till the remainder is zero. The divisor at this stage will be the required H.C.F.

To find H.C.F. of 575 and 15. Let us use Euclid's algorithm

575 = 15 × 38 + 5

Now, consider 15 and 5 and applying Euclid's algorithm again.

15 = 5 × 3 + 0

Here, the remainder is zero.

∴ H.C.F. of 15 and 5 is 5.

∴ H.C.F. of 575 and 15 is also 5.

Case 1:

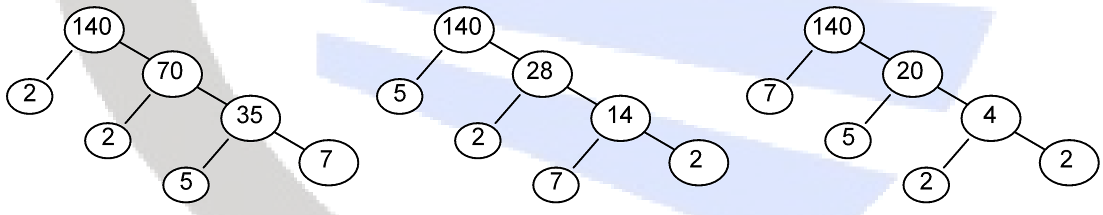
Positive integer (n)	n	$n + 2$	$n + 4$
When $n = 3q$	$3q$	(3q) + 2	$(3q) + 4$ $= 3(q + 1) + 1$

	Division by 3	divisible	leaves remainder 2 ∴ not divisible	leaves remainder 1 ∴ not divisible
Case 2:	When $n = 3q + 1$	$3q + 1$	$(3q + 1) + 2$ $= 3(q + 1)$	$(3q + 1) + 4$ $= 3(q + 1) + 2$
	Division by 3	leaves remainder 1 ∴ not divisible	divisible	leaves remainder 2 ∴ not divisible
Case 3:	When $n = 3q + 2$	$3q + 2$	$(3q + 2) + 2$ $= 3(q + 1) + 1$	$(3q + 2) + 4$ $= 3(q + 2)$
	Division by 3	leaves remainder 2 ∴ not divisible	Leaves remainder 1 ∴ not divisible	divisible

In case 1, n is divisible by 3 but $n + 2$ and $n + 4$ are not divisible by 3.
In case 2, $n + 2$ is divisible by 3 but n and $n + 4$ are not divisible by 3.
In case 3, $n + 4$ is divisible by 3 but n and $n + 2$ are not divisible by 3.
Hence, one and only one out of n, $n + 2$ or $n + 4$ is divisible by 3, where n is any positive integer.

3. FUNDAMENTAL THEOREM OF ARITHMETIC

Every composite number can be expressed as a product of primes and this expression is **unique**, except from the order in which the prime factors occur.
By taking the example of prime factorization of 140 in different orders



$140 = 2 \times 2 \times 5 \times 7$ $\qquad$ $140 = 5 \times 2 \times 7 \times 2$ $\qquad$ $140 = 7 \times 5 \times 2 \times 2$

4. IRRATIONAL NUMBERS

All real numbers which are not rational are called irrational numbers. $\sqrt{2}$, $\sqrt[3]{3}$, $-\sqrt{5}$ are some examples of irrational numbers.
There are decimals which are non-terminating and non-recurring decimal.
Example: 0.303003000300003…
Hence, we can conclude that

An irrational number is a non-terminating and non-recurring decimal and cannot be put in the form $\frac{p}{q}$ where p and q are both co-prime integers and $q \neq 0$.

5. DECIMAL REPRESENTATION OF RATIONAL NUMBERS

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of *q* is of the form $2^n \times 5^m$ where *m*, *n* are non-negative integers then *x* has a decimal representation which terminates.

For example : $0.275 = \frac{275}{10^3} = \frac{5^2 \times 11}{2^3 \times 5^3} = \frac{11}{2^3 \times 5} = \frac{11}{40}$

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of *q* is not of the form $2^m \times 5^n$, where *m*, *n* are non-negative integers, then *x* has a decimal expansion which is non-terminating repeating.

For example : $\frac{5}{3} = 1.66666...$

Rational number	Form of prime factorisation of the denominator	Decimal expansion of rational number
$x = \frac{p}{q}$, where *p* and *q* are coprime and $q \neq 0$	$q = 2^m 5^n$ where *n* and *m* are non-negative integers	terminating
	$q \neq 2^m 5^n$ where *n* and *m* are non-negative integers	non-terminating