We have learnt about various types of numbers in our earlier classes. Let us review them and know more about numbers.

Natural numbers: Counting numbers are known as natural numbers.
Thus, 1, 2, 3, 4, 5, 6, …, etc., are all natural numbers.
Whole numbers: All natural numbers together with 0 form the collection of all whole numbers.
Thus, 0, 1, 2, 3, 4, 5, 6, …, etc., are all whole numbers.
Integers: All natural numbers, 0 and negative of natural numbers form the collection of all integers.
Thus, …, –4, –3, –2, –1, 0, 1, 2, 3, 4, …, etc., are all integers.

Rational numbers: The number of the form $\frac{p}{q}$, where p and q are integers having no common factors other than 1 and $q \neq 0$, are known as rational numbers.

1.1 SIMPLEST FORM OF A RATIONAL NUMBER

A rational number $\frac{p}{q}$ is said to be in simplest form, if p and q are integers having no common factor other than 1. Here $q \neq 0$

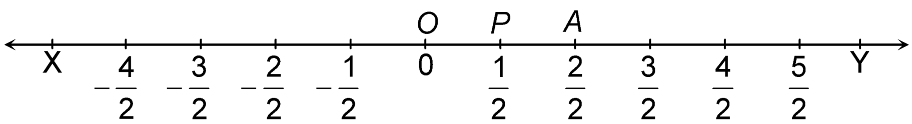
Thus, the simplest form of each of $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$ etc. is $\frac{1}{2}$.

Similarly, the simplest form of $\frac{6}{9}$ is $\frac{2}{3}$ and that of $\frac{76}{133}$ is $\frac{4}{7}$.

1.2 REPRESENTATION OF RATIONAL NUMBERS ON REAL LINE

Draw a line XY which extends endlessly in both the directions. Take a point O on it and let it represent 0 (zero).
Taking a fixed length, called unit length, mark off OA = 1 unit.
Let first we denote rational number with 2 as its denominator.

The midpoint P of OA denotes the rational number $\frac{1}{2}$. Starting from O, set off equal distances each equal to $OP = \frac{1}{2}$ unit.



From the point O, on its right, the points at distances equal to OP, $2OP$, $3OP$, $4OP$, etc., denote respectively the rational numbers $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$ etc.

Similarly, from the point O, on its left, the points at distances equal to OP, $2OP$, $3OP$, $4OP$ etc., denote respectively the rational numbers $-\frac{1}{2}, -\frac{2}{2}, -\frac{3}{2}, -\frac{4}{2}$ etc.

Thus, each rational number with 2 as its denominator can be represented by some point on the number line.

Similarly, we can denote each rational numbers with denominator 3, 4, 5, …

1.3 TO FIND RATIONAL NUMBER(S) BETWEEN GIVEN RATIONAL NUMBERS

Method 1. Let x and y be two rational numbers such that $x < y$.

Then, $\frac{1}{2}(x+y)$ is a rational number between x and y.

Method 2. Let x and y be two rational numbers such that $x < y$.

Suppose we want to find n rational numbers between x and y.

Let $d = \frac{y-x}{n+1}$.

Then, n rational numbers between x and y are:

$$(x+d), (x+2d), (x+3d), ..., (x+nd)$$

DECIMAL REPRESENTATION OF RATIONAL NUMBERS

Every rational number can be represented either as a terminating decimal or a non-terminating but repeating (recurring) decimal. For example :

Terminating: $\frac{4}{5} = 0.80$, $\frac{9}{5} = 1.8$, $\frac{5}{8} = 0.625$

and

Non-terminating but repeating: $\frac{2}{3} = 0.666...$, $\frac{1}{6} = 0.1666...$, $\frac{8}{7} = 1.142857142857...$

Thus, …, –4, –3, –2, –1, 0, 1, 2, 3, 4, …, etc., are all integers.

IRRATIONAL NUMBERS

A number 'S' is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

A real number is either rational or irrational. Thus we can say that every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

SOME USEFUL RESULTS ON IRRATIONAL NUMBERS

- Negative of an irrational number is an irrational number.
- The sum of a rational number and an irrational number is an irrational number.
- The product of a non-zero rational number and an irrational numbers is an irrational number.
- The sum, difference, product and quotient of two irrational numbers need not be an irrational number.

CONVERSION OF REPEATING DECIMAL INTO RATIONAL NUMBER

4.1 CONVERSION OF PURE NON-TERMINATING AND REPEATING DECIMAL INTO A RATIONAL NUMBER $\frac{p}{q}$ FORM.

1. Put the given decimal equal to x.
2. Find out the number of repeating digits under Dots or Bar.
3. Multiply by 10, 100, … and so on respectively according to the repeating digits are 1, 2, …. and so on.
4. Subtract step 1 from step 3.
5. Divide both sides of the resulting equation by coefficient of *x*.
6. Write the rational number thus obtained in the simplest $\left(\frac{p}{q}\right)$ form.

4.2 CONVERSION OF MIXED NON-TERMINATING AND REPEATING DECIMAL INTO A RATIONAL NUMBER $\frac{p}{q}$ FORM.

1. Take the mixed recurring decimal and put it equal to x.
2. Count the number of non-recurring digits after the decimal point. Let it be *a*.
3. Multiply both sides of *x* by 10^a so that only the repeating decimal is on the right hand side of the decimal point.
4. Multiply both sides of *x* by 10^{a+b} where *b* is the number of repeating digits in the decimal part.
5. Subtract the number in step 3 from number in step 4.
6. Divide both sides of the resulting equation by the coefficient of *x*.
7. Write the rational number thus obtained in the simplest $\left(\frac{p}{q}\right)$ form.

REAL NUMBER AND REAL NUMBER LINE

5.1 EXISTENCE OF SQUARE ROOT OF A POSITIVE REAL NUMBER

For any positive real number *x*, we have

$$\sqrt{\left(\frac{x+1}{2}\right)^2-\left(\frac{x-1}{2}\right)^2}=\sqrt{\frac{x^2+2x+1}{4}-\frac{x^2-2x+1}{4}}=\sqrt{\frac{4x}{4}}=\sqrt{x}$$

To find the positive square root of a positive real number, we follow the following steps.

1. Obtain the positive real number *x* (say).
2. Draw a line and mark a point *A* on it.
3. Mark a point *B* on the line such that *AB* = *x* units.
4. From point *B* mark a distance of 1 unit and mark this new point as *P*.
5. Find the mid-point of *AP* and mark the point as *O*.
6. Draw a circle with centre *O* and radius *OP*.
7. Draw a line perpendicular to *AP* passing through *B* and intersecting the semicircle at *D*. Length *BD* is equal to $\sqrt{x}$.

5.2 REPRESENTATION OF NUMBERS ON THE NUMBER LINE BY MEANS OF MAGNIFYING GLASS

The process of visualization of numbers on the number line through a magnifying glass is known as successive magnification.

Sometimes, we are unable to check the numbers like 2.849 and $6.\overline{24}$ on the number line, we seek the help of magnifying glass by dividing the part into subparts again and again to ensure the accuracy of the given number.

5.2.1 Method to find such numbers on the number line

1. Choose the two consecutive integral numbers in which the given number lies.
2. Choose the two consecutive decimal points in which the given decimal part lies by dividing the two given decimal parts into required equal parts.
3. Visualize the required number through magnifying glass.

LAWS OF EXPONENTS FOR REAL NUMBERS

In the language of exponents, we define $\sqrt[n]{a}=a^{\frac{1}{n}}$. So, in particular, $\sqrt[3]{5}=5^{\frac{1}{3}}$.

There are now two ways to look at $16^{\frac{3}{2}}$.

$\Rightarrow \quad [16^{\frac{1}{2}}]^3=[4]^3 \quad \Rightarrow \quad 64$

$\Rightarrow \quad 16^{\frac{3}{2}}=(16^3)^{\frac{1}{2}}=(4096)^{\frac{1}{2}}=64$

Therefore, we have the following definition:

Let $a > 0$ be a real number. Let r and s be integers such that r and s have no common factors other than 1, and $s > 0.$ Then,

$$a^{\frac{r}{s}} = \left(\sqrt[s]{a}\right)^r = \sqrt[s]{a^r}$$

We now have the following laws of exponents :

Let a > 0 be a real number and p and q be rational numbers. Then, we have

(i) $a^p.a^q = a^{p+q}$

(ii) $(a^p)^q = a^{pq}$

(iii) $\frac{a^p}{a^q} = a^{p-q}$

(iv) $a^p b^p = (ab)^p$