INTRODUCTION

Greek mathematicians Thales and Pythagoras discovered a number of geometric properties.
Around 300 *BC* all known work in the field of geometry was collected in book form by Euclid, a teacher of mathematics at Alexandria in Egypt. The collected work had a great impact all over the world for understanding geometry as a subject in a systematic manner.
We shall discuss Euclid's technique called Euclid's Geometry to establish and examine various properties of Geometry.

1.1 EUCLID'S DEFINITIONS

Points : A point is that which has no part.
Line : A line is breadthless length.
Surface : A surface is that which has length and breadth only.
Ends of a line : The ends of a line are points.
Edges of a Surface : The edges of a surface are lines.
Straight line : A straight line is a line which lies evenly with the points on itself.
Plane Surface : A plane surface is a surface which lies evenly with the straight lines on itself.

1.2 EUCLID'S AXIOMS

Axioms : The assumptions, used throughout in mathematics which are obvious universal truths, are termed as axioms.
Axioms given by Euclid are as under :

1. Things which are equal to the same thing are equal to one another.
   i.e., if $a = c$ and $b = c$, then $a = b$.
   For example, if area of a circle is equal to that of square and the area of the square is equal to that of a rectangle, then the area of the circle is equal to the area of the rectangle.
2. If equals are added to equals, the wholes are equal.
   i.e., if $a = b$ and $c = d$, then $a + c = b + d$.
   Also $a = b \Rightarrow a + c = b + d$.
   Here, a, b, c and d are same kind of things.
3. If equal are subtracted from equals, the remainders are equal.
4. The things which coincide with one another are equal.
5. The whole is greater than the part.
   i.e. if $a > b$, then there exists c such that $a = b + c$.
   Here, b is a part of a and therefore, a is greater than b.
6. Things which are double the same things are equal to one another.
7. Things which are halves of the same things are equal to one another.

1.3 EUCLID'S POSTULATES

Postulates : The assumptions, specific to geometry which are obvious universal truths, are termed as postulates. Euclid gave five postulates as stated below :
Postulate 1 : A straight line may be drawn from any one point to any other point.
Postulate 2 : A terminated line (i.e., a line segment) can be produced indefinitely on either side.

Postulate 3 : A circle can be drawn with any centre and any radius.

Postulate 4 : All right angles are equal to one another.

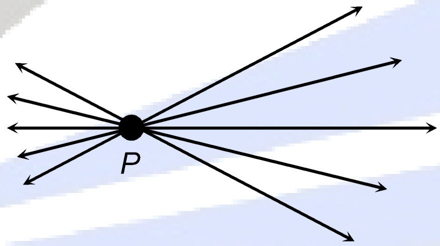
Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

1.4 INCIDENCE AXIOMS

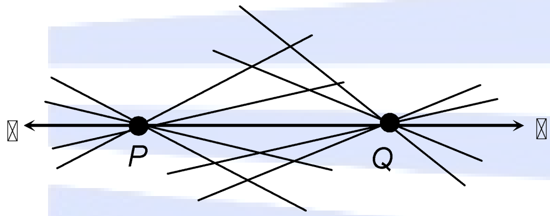
Here, we shall assume some properties about lines and points without any proof but these properties are obvious universal truths. These properties are taken as axioms.

Axiom 1: A line contains infinitely many points.

Axiom 2 : Through a given point, infinitely many lines can be made to pass. In figure, infinitely many lines pass through the point *P*.



Axiom 3 : Given two distinct points, there exists one and only one line through them.



In figure, we observe that, out of all lines passing through the point *P* there is exactly one line ‘✦ ‘ which also passes through *Q*. Similarly, out of all lines passing through the point *Q* there is exactly one line ‘✦ ’ which also passes through *P*. Hence, we find exactly one line ‘✦’ which can be drawn through two points *P* and *Q*.

To determine a particular line, only two distinct points of the line are needed. A line ‘✦’ which passes through two distinct points *P* and *Q* is denoted by *PQ* or by $\overline{PQ}$.

1. Three or more points are collinear if one and only one line can be made to pass through these points.

Three or more points are non-collinear if they are not collinear.

2. Three or more lines are said to be concurrent if they all pass through a unique point. The point is called the point of concurrence for the lines.

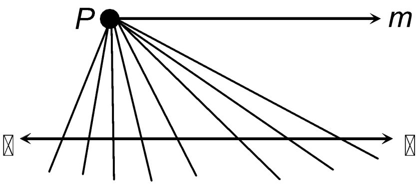
1.5 EQUIVALENT VERSIONS OF EUCLID’S FIFTH POSTULATE

There are several equivalent versions of the fifth postulate of Euclid. One such version is stated as ‘Playfair’s Axiom’ which is given below :

1.5.1 Playfair’s Axiom (Axiom for Parallel Lines)

For every line ℓ and for every point P not lying on ℓ, there exists a unique line m passing through P and parallel to ℓ.

Let us observe it in figure.



Another version of the above axiom is as stated below :

Two distinct intersecting lines cannot be parallel to the same line.

In figure, there are infinitely many straight lines through *P* but there is exactly one line *m* which is parallel to ℓ. Thus, two intersecting lines cannot be parallel to the same line.

1.6 A STATEMENT AS A THEOREM

Euclid deduced 465 geometrical properties by using definitions, axioms and postulates. These properties are called theorems.

The properties confirmed through logical reasoning based on some axioms, postulates and previously proved results are called theorems or propositions.

1.6.1 Two distinct lines cannot have more than one point in common.

Let us suppose that the two lines intersect in two distinct points, say *P* and *Q*. So, we have two lines passing through two distinct points *P* and *Q*. But this assumption contradicts with the axiom that only one line can pass through two distinct points. Hence our assumption is wrong. Thus we conclude that two distinct lines cannot have more than one point in common.

1.7 A CONSISTENT COLLECTION OF AXIOMS

A collection of axioms is consistent, if it is impossible to derive from these axioms a result that contradicts any axiom or postulate or previously proved property.

All the attempts to prove Euclid’s fifth postulate using the first 4 postulates failed. But they led to the discovery of several other geometries, called non-Euclidean geometries.