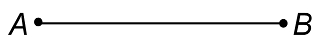
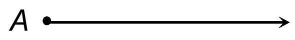
1. BASIC TERMS AND DEFINITIONS

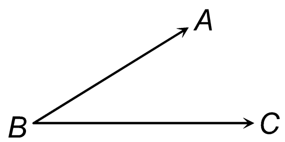
Line Segment : The part of a line with two end points is called a line segment.



Ray : The part of a pair of line with one end point is called a ray



Angle : An angle is the union of two non-collinear rays with a common initial point.



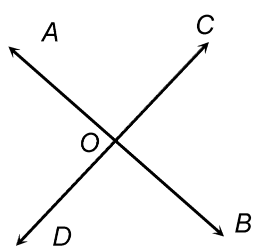
1.1 TYPES OF ANGLES

Acute angle : An angle measures between 0° and 90°.
Obtuse angle : An angle measures between 90° and 180°.
Right angle : An angle measures exactly equal to 90°.
Straight angle : An angle measures exactly equal to 180°.
Reflex angle : An angle measures between 180° and 360°.
Adjacent angle : Two angles are said to the adjacent angles if they have same vertex, a common arm and are non-overlapping.
Complementary angles : Two angles whose sum is 90° are called complementary angles.
Supplementary angles : Two angles whose sum is 180° are called supplementary angles.
Vertically opposite angles : The pair of angles formed when two lines intersect at a point.
Collinear points : Three or more points are said to be collinear if they lie on the same line.

1.2 PAIRS OF ANGLES

Axiom 1 : If a ray stands on a line, then the sum of two adjacent angles so formed is 180º.
When the sum of two adjacent angles is 180º, then they are called as **linear pair of angles.**
Axiom 2 : If the sum of two adjacent angles is 180º, then the non-common arms of the angles form a line.
The two axioms above together is called the **Linear Pair Axiom.**
Theorem 1 : If two lines intersect each other then the vertically opposite angles are equal.

Given : Two lines *AB* and *CD* intersect at a point *O*.
To prove :
(i) $\angle AOC = \angle DOB$
(ii) $\angle BOC = \angle AOD$



Proof : Since ray *OD* stands on line *AB*
$\therefore$ $\angle AOD + \angle BOD = 180°$ …(i) [linear pair]

Again, ray *OA* stands on line *CD*
$\therefore$ $\angle AOD + \angle AOC = 180°$ …(ii) [linear pair]
From equation (i) and (ii), we have
$\angle AOD + \angle BOD = \angle AOD + \angle AOC$

⇒ $\angle BOD = \angle AOC$

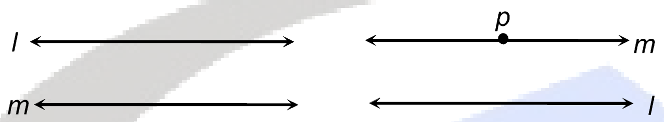
Thus, $\angle AOC = \angle DOB$

Similarly we can prove that

$\angle BOC = \angle AOD$

2. PARALLEL LINES AND THEIR PROPERTIES

Parallel lines : Two lines in a plane, that is, the co-planar lines whose intersection is empty are said to be parallel lines

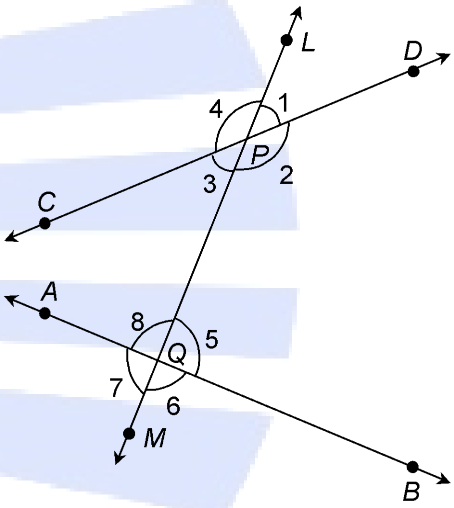


2.1 ANGLES MADE BY A TRANSVERSAL WITH TWO LINES

Definition:

Transversal : A line which intersects two or more given lines at distinct points, is called a **"Transversal"** of the given lines.

In the adjoining figure, *AB* and *CD* are given two lines and a transversal *LM* intersects them at points *P* and *Q* respectively.



Corresponding angles : Two angles on the same side of a transversal are known as the **corresponding angles** if both lies either above the two lines or below the two lines.

For example

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 4$ and $\angle 8$

(iii) $\angle 2$ and $\angle 6$ (iv) $\angle 3$ and $\angle 7$

Alternate interior angles : The following pairs of angles are called the pairs of alternate interior angles.

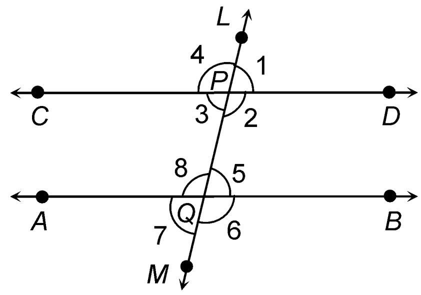
For example

(i) $\angle 3$ and $\angle 5$ (ii) $\angle 2$ and $\angle 8$

Consecutive interior angles : The pairs of interior angles on the same side of the transversal are called pairs of consecutive interior angles.

For example

(i) $\angle 2$ and $\angle 5$ (ii) $\angle 3$ and $\angle 8$



Axiom 3 : If a transversal intersects two parallel lines, then each pair pf corresponding angles is equal.

Axiom 4 : If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.

Theorem 2 : If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

Theorem 3 : If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.

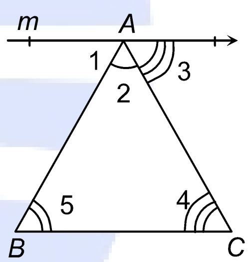
Theorem 4 : If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

Theorem 5 : If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary then the two lines are parallel.

2.2 LINES PARALLEL TO THE SAME LINE

Theorem 6 : Lines which are parallel to the same line are parallel to each other.

3. ANGLE SUM PROPERTY OF A TRIANGLE



Theorem 7 : Angles sum property of a triangle.

The sum of three angles of a triangle is 180.

Given : A triangle *ABC*

To Prove : $\angle 2 + \angle 5 + \angle 4 = 180°$

Construction : Through *A*, draw a line '*m*' parallel to *BC*

Proof : Since *m* || *BC* and transversal *AB* intersects at *A* and *B* respectively,

therefore we have

$\angle 1 = \angle 5$ …(i) [alternate angles]

Similarly, *m* || *BC* and the transversal *AC* intersects them at *A* and *C* respectively.

So, $\angle 3 = \angle 4$ …(ii) [alternate angles]

Adding equation (i) and (ii), we have

$\angle 1 + \angle 3 = \angle 5 + \angle 4$

Adding $\angle 2$ to both sides, we have

$\angle 1 + \angle 2 + \angle 3 = \angle 5 + \angle 4 + \angle 2$

But, $\angle 1 + \angle 2 + \angle 3 = 180°$ [Linear pair]

Hence, $\angle 2 + \angle 5 + \angle 4 = 180°$ [proved]

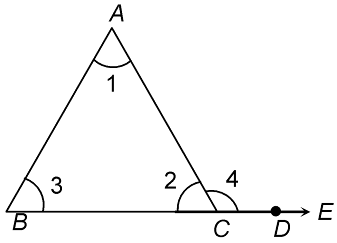
Theorem 8 : Exterior angle theorem

If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Given : A triangle *ABC*. *D* is a point on *BC* produced, forming exterior $\angle ACE$.

To prove : $\angle ACD = \angle BAC + \angle ABC$

Proof : In ΔABC



We have $\angle 1 + \angle 2 + \angle 3 = 180°$ [Angle sum property] …(i)

Also $\angle 2 + \angle 4 = 180°$ [linear pair] …(ii)

From (i) and (ii), we get

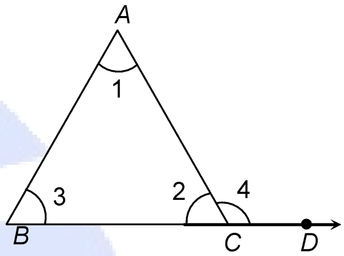
$\angle 1 + \angle 2 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow$ $\angle 1 + \angle 3 = \angle 4$

Hence $\angle 4 = \angle 1 + \angle 3$

or $\angle ACD = \angle BAC + \angle ABC$

Corollary 1 : An exterior angle of a triangle is greater than either of the interior opposite angles.



Proof : Let ABC be a triangle whose side BC is produced to form exterior angle $\angle 4$

Thus $\angle 1 + \angle 3 = \angle 4$

$\Rightarrow$ $\angle 4 > \angle 1$ and $\angle 4 > \angle 3$