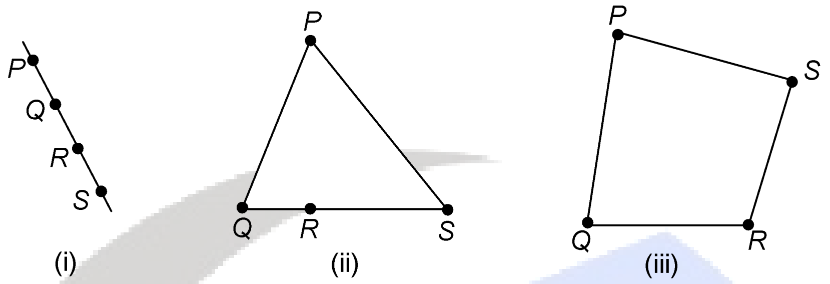
1. INTRODUCTION

When three non-collinear points on a sheet of paper are joined in pairs, the closed figure so obtained is a triangle.

Now take four distinct points on a sheet of paper and join these points in pairs. The possible type of figure made are as below :



Case 1. When all the four points are collinear as shown in figure (i) we get a line segment. Here in figure, line segment *PS* is made.

Case 2. When three points out of the four points are collinear, we get a triangle as shown in figure (ii).

Case 3. When no three points out of the four points are collinear, we get a closed figure with four sides, four angles and four vertices as shown in figure (iii). Such closed figure is called quadrilateral.

A closed figure having four sides, four angles and four vertices is called a **quadrilateral**.

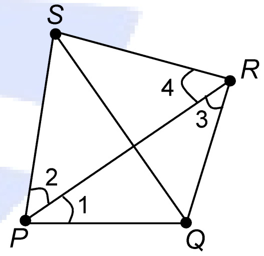
1.1 ANGLE SUM PROPERTY OF A QUADRILATERAL

Theorem 1 : The sum of the four angles of a quadrilateral is 360°.

We have quadrilateral *PQRS*. *PR* is its one diagonal.

From ΔPQR,

$\angle 1 + \angle 3 + \angle Q = 180°$ …(i)



From ΔPSR,

$\angle 2 + \angle 4 + \angle S = 180°$ …(ii)

Adding (i) and (ii), we have

$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + \angle Q + \angle S = 180° + 180°$

$\Rightarrow \quad \angle QPS + \angle QRS + \angle Q + \angle S = 360°$

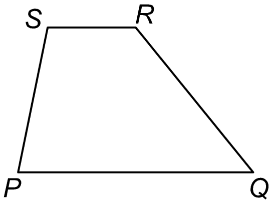
$\Rightarrow \quad \angle P + \angle R + \angle Q + \angle S = 360°$

i.e., $\quad \angle P + \angle Q + \angle R + \angle S = 360°$

1.2 TYPES OF QUADRILATERALS

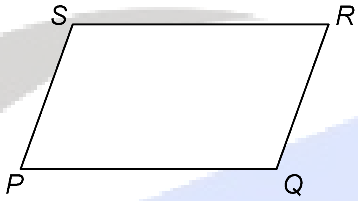
1.2.1 Trapezium

If one pair of opposite sides of a quadrilateral are parallel, then the quadrilateral is called a trapezium. In figure *PQ* || *RS* and therefore, the quadrilateral *PQRS* is a trapezium.



1.2.2 Parallelogram

If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram. In figure $PQ \parallel RS$ and $QR \parallel PS$.

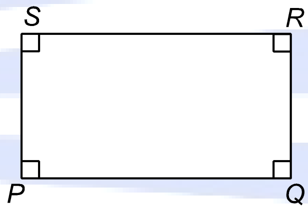


Therefore, the quadrilateral *PQRS* is a parallelogram.

1.2.3 Rectangle

In a parallelogram, if one angle is right angle, then the quadrilateral is called a rectangle.

In figure $PQ \parallel SR$, $PS \parallel RQ$ and $\angle P = \angle Q = \angle R = \angle S = 90°$

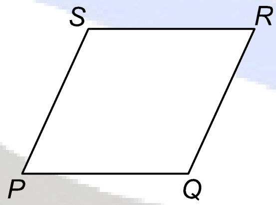


Here, quadrilateral *PQRS* is a rectangle.

1.2.4 Rhombus

In a parallelogram if all the four sides be equal, then the quadrilateral is a rhombus.

In figure $PQ \parallel SR$, $PS \parallel RQ$ and $PQ = SR = PS = RQ$



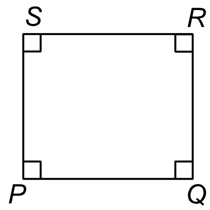
Here, quadrilateral *PQRS* is a rhombus.

1.2.5 Square

In a quadrilateral if all the four sides are equal and one angle is right angle, then all the other three angles are also right angles. Such a quadrilateral is called a square.

In figure $PQ = SR = PS = RQ$

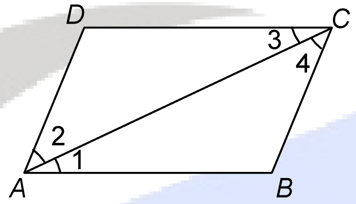
and $\angle P = \angle Q = \angle R = \angle S = 90°$



Here, quadrilateral *PQRS* is a square.

Theorem 2 : A diagonal of a parallelogram divides it into two congruent triangles.

Given : *AC* is a diagonal of the parallelogram *ABCD* as shown in figure.



To prove : $\Delta ABC \cong \Delta CDA$.

Proof : We have $AB \parallel CD$ [Opposite sides of parallelogram *ABCD*]

$\Rightarrow$ $\angle 1 = \angle 3$ ...(i) [Alternate angles with transversal *AC*]

Now, $BC \parallel AD$ [Opposite sides of parallelogram *ABCD*]

$\Rightarrow$ $\angle 2 = \angle 4$ ...(ii) [Alternate angles with transversal *AC*]

In ΔABC and ΔCDA, we have

$\angle 1 = \angle 3$ [From (i)]

$\angle 2 = \angle 4$ [From (ii)]

and $AC = CA$ [Common]

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By ASA Congruence]

Hence the theorem is proved.

1.3 NECESSARY CONDITIONS OF A PARALLELOGRAM

Theorem 3 : In a parallelogram, opposite sides are equal.

Theorem 4 : In a parallelogram, opposite angles are equal.

Theorem 5 : The diagonals of a parallelogram bisect each other.

1.4 SUFFICIENT CONDITIONS FOR A QUADRILATERAL TO BE PARALLELOGRAM

Theorem 6 : (Converse of Theorem 2). If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.

Theorem 7 : (Converse of Theorem 3). If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

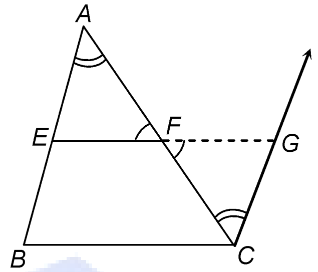
Theorem 8 : (Converse of Theorem 4). If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Theorem 9 : A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

2. MID POINT THEOREM FOR A TRIANGLE

Theorem 10 : The line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

Given : E and F are the mid-points of the sides AB and AC respectively of the ΔABC (figure).



To prove : $EF \parallel BC$ and $EF = \frac{1}{2} BC$

Construction : Through the vertex C, CG is drawn parallel to AB and it meets EF (produced) at G.

Proof : In ΔCGF and ΔAEF, we have

$AF = CF$ [∵ F is mid-point of AC]

$\angle AFE = \angle CFG$ [Vertically opposite angles]

$\angle EAF = \angle GCF$ [Alternate angles]

Therefore, $\Delta AEF \cong \Delta CGF$ [By SAS congruence]

⇒ $AE = CG$ …(i) [cpct]

$EF = FG$ …(ii) [cpct]

Also, $AE = BE$ …(iii) [∵ E is mid-point of AB]

From (i) and (ii), $BE = CG$

Also, by construction $BE \parallel CG$

Therefore, $BCGE$ is a parallelogram.

⇒ $EG \parallel BC$

⇒ $EF \parallel BC$

⇒ $BC = EG = EF + FG = EF + EF$ [From (ii)]

$= 2\,EF$

⇒ $2\,EF = BC$

⇒ $2\,EF = BC$

⇒ $EF = \frac{1}{2} BC$

Hence, the theorem is proved.

Theorem 11 : (Converse of theorem 10) The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.