1. SEQUENCE AND SERIES

Sequence: A sequence is an arrangement of number in a definite order, according to a definite rule.

Terms: Various numbers occurring in a sequence are called terms or element.

Consider the following lists of number:

3, 6, 9, 12, ……………

4, 8, 12, 16, ……………

-3, -2, -1, 0, ……………

In all the list above, we observe that each successive terms are obtained by adding a fixed constant to the preceding terms. Such list of numbers is said to form on **Arithmetic Progression (AP).**

Arithmetic Progression: An arithmetic progression is a sequence in which the difference between any term and its just preceding term is constant throughout.

This constant is called the **common difference (d)** of the A.P.

Common difference can be positive, negative or zero.

or

An Arithmetic Progression is a sequence whose terms increase or decrease by a fixed number. This fixed number is called common difference of an arithmetic Progression.

Let us denote first term of A.P. by a or t, second term by a_2 or t_2 and nth term by a_n or t_n & the common difference by d. Then the A.P. becomes

$a_1, a_2, a_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots a_n$

where $a_2 - a_1 = d$

or $a_2 = a_1 + d$

similarly $a_3 = a_2 + d$

Note :

(i) If n^{th} term of any sequence is linear expansion in n then sequence is an Arithmetic Progression.

Example : If $a_n = 2n + 3$ tnen

for $n = 1$	for $n = 2$	for $n = 3$	
$a_1 = 5$	$a_2 = 7$	$a_3 = 9$	…..

Hence $a_1, a_2, a_3,$ …. Are in Arithmetic Progression because they have the common difference 2.

(ii) The coefficient of n in the n^{1h} term of linear expression denotes the common difference of an Arithmetic Progression.

(iii) If sum of n terms of any sequence in quadratic expression in n (i.e., $S_n = an^2 + bn + c$) the sequence is an Arithmetic Progression.

If the sum of $(n-1)$ terms is substracted by sum of n terms then n^{th} term can be calculated.

$S_n - S_{n-1} = t_n$

$\Rightarrow \quad (an^2 + bn + c) - (a(n-1)^2 + b(n-1) + c) = t_n$

$\Rightarrow \quad (an^2 + bn + c) - (an^2 + bn + c + a - 2an - b) = t_n$

$\Rightarrow \quad 2an + b - a = t_n$

Here n^{th} term is linear expression in 'n' hence it is an Arithmetic Progression with common difference $2a$.

Thus

$a, a + d, a + 2d$, …………………..

forms an A.P. whose first term is 'a' & common difference is 'd'

This is called general form of an A.P.

Finite A.P.: An A.P. containing finite number of terms is called finite A.P.

e.g. 147, 149, 151 ………………….. 163.

Infinite A.P.: An A.P. containing infinite terms is called infinite A.P.

e.g. 6, 9, 12, 15 ………………………..

2. nth TERM OF AN A.P.

Let a_1, a_2, a_3 …….. be an A.P., with first term as a, and common difference as d.

First term is $= a$ (i)

Second term (a_2) $= a + d$ (ii)

$= a + (2-1)d$

Third term (a_3) $= a_2 + d$ (iii)

$= a + d + d$ [from (i)]

$= a + 2d$

or $= a + (3-1)d$

Fourth term $a_4 = a_3 + d$

or $= a + 2d + d$ [from (iii)]

$= a + 3d$

$= a + (4-1)d$

∴ nth term $a_n = a + (n-1)d$

a_n is also called as general term of an A.P.

If there are P terms in the A.P. then a_p represents the last term which can also be denoted by l.

2.1 TO FIND nth TERM FROM THE END OF AN A.P.

Consider the following A.P. $a, a+d, a+2d, \dots.(l-2d), (l-d)$, l

where l is the last term

last term $l = l - (1-1)d$

2nd last term $l - d = l - (2-1)\,d$

3rd last term $l - 2d = l - (3-1)d$

…………………………….

…………………………….

2.2 CONDITION FOR TERMS TO BE IN A.P.

If three numbers *a, b, c,* in order are in A.P. Then,

$b - a =$ common difference $= c - b$

⇒ $b - a = c - b$

⇒ $2b = a + c$

3. SUM OF n TERMS OF AN A.P.

Let *a* be the first term and *d* be the common difference of an A.P. l is the last term where $l = a + (n-1)\,d$.

Sum of first n terms of the given A.P. is given by

$$S_n = a + (a+d) + (a+2d) + \dots.(l-2d) + (l-d) + l \qquad \dots(i)$$

Writing in reverse order

$$S_n = l + (l-d) + (l-2d) + \dots.(a+2d) + (a+d) + a \qquad \dots(ii)$$

Adding (i) and (ii) we get

$$2S_n = \underbrace{(a+l) + (a+l) + (a+l) + \dots + (a+l)}_{n \text{ times}}$$

$$2S_n = n(a+l)$$

$S_n = \frac{n}{2}(a + l)$

$= \frac{n}{2}[a + a + (n-1)d]$ [∵ $l = a + (n-1)d$]

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}(a + a_n)$ where $a_n = a + (n-1)d$

Some important properties :

(i) If a_1, a_2, a_3 ….. are in arithmetic progression then

$a_1 \pm k, a_2 \pm k, a_3 \pm k$ ….. are also in arithmetic progression.

(ii) If a_1, a_2, a_3 …. are in arithmetic progression then

$a_1 k, a_2 k, a_3 k$ … and $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}$ ….. are also in arithmetic progression ($k \neq 0$).

(iii) If a_1, a_2, a_3 … and b_1, b_2, b_3 ….. are two arithmetic progressions then

$a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3$ ….. are also in an arithmetic progressions.

(iv) If a_1, a_2, a_3 …. And b_1, b_2, b_3 ….. are two arithmetic progressions then

$a_1 b_1, a_2 b_2, a_3 b_3 ....$ and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$ ….. are not in arithmetic progression.

(v) Selection of Terms in Arithmetic progressions

Some times certain number of terms in A.P. are required. The following ways of selecting terms are convenient.

Number of terms	Terms	common difference
3	$a - d, a, a + d$	d
4	$a - 3d, a - d,, a + d, a + 3d$	$2d$
5	$a - 2d, a - d, a, a + d, a + 2d$	d
6	$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$	$2d$