

Secant: A line, which intersects a circle in two distinct points, is called a secant.

Tangent: A line meeting a circle only in one point is called a tangent to the circle at that point.

The point at which the tangent line meets the circle is called the point of contact.

Length of tangent: The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.

Theorem 1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given : A circle with centre O and a tangent AB at a point P of the circle.

To prove : $OP \perp AB$

Construction :

Take a point Q on AB . Join OQ .

Proof : Q is a point on the tangent AB , other than the point of contact P .

$\therefore Q$ lies outside the circle.

Let OQ intersect the circle at R .

Then, $OR < OQ$

..... (i)

But, $OP = OR$

[radii of the same circle]. (ii)

From (i) and (ii)

$\therefore OP < OQ$

OP is the shortest distance between the point O and the line AB .

But, the shortest distance between a point and a line is the perpendicular distance.

$\therefore OP \perp AB$.

Theorem 2 : The lengths of tangents drawn from an external point to a circle are equal.

Given : Two tangents AP and AQ are drawn from a point A to a circle with centre O .

To prove : $AP = AQ$

Construction : Join OP , OQ and OA .

Proof : AP is a tangent at P and OP is the radius through P .

$\therefore OP \perp AP$

Similarly, $OQ \perp AQ$

In the right $\triangle OPA$ and OQA , we have

$$OP = OQ$$

[radii of the same circle]

$$OA = OA$$

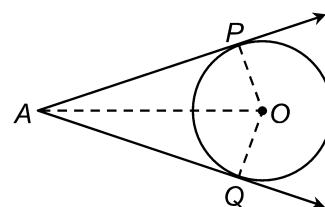
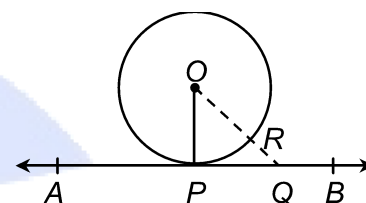
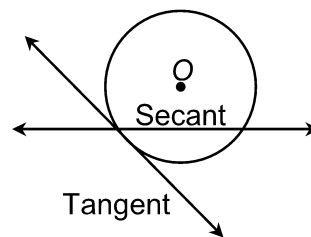
[common]

$$\angle OPA = \angle OQA$$

[both 90°]

$$\therefore \triangle OPA \cong \triangle OQA$$

[by RHS-congruence]



Hence, $AP = AQ$.

[cpct]

Also $\angle OAP = \angle OAQ$

[cpct]

\therefore OA is the angle bisector of $\angle PAQ$.

