1. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

An equation is a statement in which there is an equality sign between two algebraic expressions.

For ex : $2x + 5 = x + 3$

$3x - 4 = 20$ etc.

Some basic facts about the linear equation are as follows :

- The equation of the form $ax = b$ or $ax + b = 0$, where a and b are two real numbers such that $a \neq 0$ and x is a variable is called a linear equation in one variable.
- The general form of a linear equation in two variable is $ax + by + c = 0$ or $ax + by = c$ where a, b, c are real numbers and $a \neq 0$, $b \neq 0$ and x, y are variables.
- The graph of a linear equation in two variables is a straight line.
- The graph of a linear equation in one variable is a straight line parallel to x-axis for $ay = b$ and parallel to y-axis for $ax = b$.
- A pair of linear equations in two variables is said to form a system of simultaneous linear equations.
- The value of the variable x and y satisfying each one of the equations in a given system of linear equations in x and y simultaneously is called a solution of the system.

2. GRAPHICAL METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

A pair of linear equation can be solved graphically. As it shows a pair of straight lines, it may be parallel, or may coincide or may intersect. To solve these equations graphically, follow some basic rules :

- Read the problem carefully to find the unknowns (variables) which are to be calculated.
- Depict the unknowns by x and y etc.
- Use the given conditions in the problem to make equations in unknown x and y.
- Make the proper tables for both the equations.
- Draw the graph of both the equations on the same set of axis.
- Locate the co-ordinates of point of intersection of the graph, if any.
- Coordinates of point of intersection will give us the required solution.

3. SYSTEM OF TWO SIMULTANEOUS LINEAR EQUATIONS IN x AND y

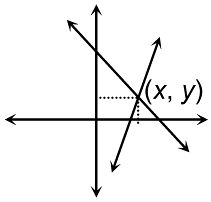
Consistent system: A system of two linear equations is said to be consistent if it has at least one solution.

Inconsistent system: A system of two linear equations is said to be inconsistent if it has no solution.

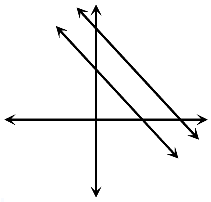
Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is a system of two linear equations.

The following cases occur :

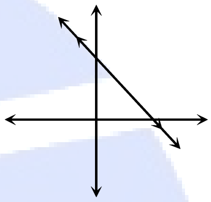
(i) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, it has a unique solution. The graph of lines intersect at one point. The system is consistent



(ii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ .It has no solution. The graph of both lines are parallel to each other. The system is inconsistent



(iii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .It has infinite many solutions. Every solution of one equation is a solution of other also. The graph of both equations are coincident lines. The system is dependent consistent.



4. ALGEBRAIC METHODS OF SOLVING LINEAR EQUATION

4.1 ELIMINATION METHOD

Eliminating One variable by making the coefficient equal to get the value of other variable and than put it in any equation to find other variable.

- First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.
- Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to step 3.
- Solve the equation in one variable (x or y) so obtained to get its value.
- Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

4.2 SUBSTITUTION METHOD

Find the value of any one variable in terms of other and than use it to find other variable from the second equation.

· Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.

· Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved.

· Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

4.3 COMPARISON METHOD

Find the value of one variable from both the equation and equate them to get the value of other variable.

Let any pair of linear equations in two variables is of the form

$$a_1x + b_1y + c_1 = 0 \quad \ldots(i)$$

$$a_2x + b_2y + c_2 = 0 \quad \ldots(ii)$$

- Find the value of one variable, say *y* in terms of other variable, i.e. *x* from equation (i), to get equation (iii).
- Find the value of the same variable (as in step 1) in terms of other variable from equation (ii) to get equation (iv).
- By equating the variable from equation (iii) and (iv) obtained in above two steps. We get the value of second variable.
- Substituting the value of above said variable in equation (iii), we get the value of another variable.

4.4 CROSS MULTIPLICATION METHOD

Let the equation

$$a_1x + b_1y + c_1 = 0 \quad \ldots(i)$$

$$a_2x + b_2y + c_2 = 0 \quad \ldots(ii)$$

To obtain the values of *x* and *y,* we follow these steps:

- Multiply Equation (i) by b_2 and (ii) by b_1, to get

$$b_2a_1x + b_2b_1y + b_2c_1 = 0 \quad \ldots(iii)$$

$$b_1a_2x + b_1b_2y + b_1c_2 = 0 \quad \ldots(iv)$$

- Subtracting Equation (iv) from (iii), we get:

$$(b_2a_1 - b_1a_2)x + (b_2b_1 - b_1b_2)y + (b_2c_1 - b_1c_2) = 0$$

i.e. $(b_2a_1 - b_1a_2)x = b_1c_2 - b_2c_1$

So, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, if $a_1b_2 - a_2b_1 \neq 0$ …(v)

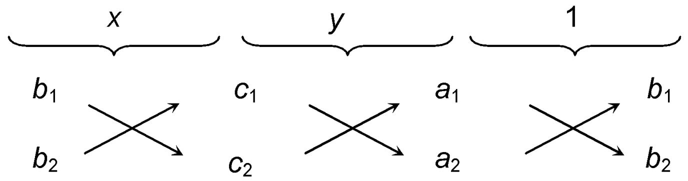
- Substituting this value of *x* in (i) or (ii), we get

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad \ldots(vi)$$

Note that you can write the solution given by equations (v) and (vi) in the following form:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad \ldots(vii)$$

In remembering the above result, the following diagram may be helpful to you:



…(viii)

For solving a pair of linear equations by this method, we will follow the following steps:

- Write the given equations in the form (i) and (ii).
- Taking the help of the diagram above, write equations as given in (vii).
- Find x and y.

5. EQUATIONS REDUCIBLE TO A PAIR OF LINEAR EQUATIONS

We shall discuss the solution of such pairs of equations which are not linear but can be reduced to linear form by making some suitable substitutions.

6. APPLICATIONS TO WORLD PROBLEMS

- Read the problem carefully and identify the unknown quantities. Give these quantities a variable name like x, y, u, v etc.
- Identify the variables to be determined.
- Read the problem carefully and formulate the equations in terms of the variables to be determined.
- Solve the equations obtained in step 3 using any one of the methods used earlier.