1. POLYNOMIALS

An expression $p(x)$ of the form $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n$ where all $a_0, a_1, a_2, ..., a_n$ are real numbers and *n* is a non-negative integer, is called a polynomial.

The degree of a polynomial in one variable is the greatest exponent of that variable.

$a_0, a_1, a_2 ..., a_n$ are called the co-efficient of the polynomial $p(x)$.

a_n is called constant term.

Example : $p(x) = 3x - 2$

$q(y) = 3y^2 - 2y + 4$

1.1 DEGREE OF A POLYNOMIAL

The exponent of the term with the highest power in a polynomial is known as its degree.

$f(x) = 8x^3 - 2x^2 + 8x - 21$ and $g(x) = 9x^2 - 3x + 12$ are polynomials of degree 3 and 2 respectively.

Thus, $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is a polynomial of degree *n*, if $a_0 \neq 0$.

On the basis of degree of a polynomial, we have following standard names for the polynomials.

A polynomial of degree zero is called a constant polynomial; $f(x) = 9$.

A polynomial of degree 1 is called a linear polynomial.

Example: $2x + 3, \frac{1}{3}u + 7$ etc.

A polynomial of degree 2 is called a quadratic polynomial.

Example: $x^2 + 2x + 3$, $y^2 - 9$ etc.

A polynomial of degree 3 is called a cubic polynomial.

Example: $x^3 + 7x - 3$, $-x^3 + x^2 + \sqrt{3}x$ etc.

A polynomial of degree 4 is called a biquadratic polynomial.

Example: $3u^4 - 5u^3 + 2u^2 + 7$.

1.2 VALUE OF A POLYNOMIAL

If $f(x)$ is a polynomial and α is any real number, then the real number obtained by replacing *x* by α in $f(x)$ is called the value of $f(x)$ at *x* = α and is denoted by $f(\alpha)$.

e.g. : Value of $p(x) = 5x^2 - 3x + 7$ at $x = 1$ will be

$\therefore \quad p(1) = 5(1)^2 - 3(1) + 7 = 5 - 3 + 7 = 9$

1.3 ZEROS OF A POLYNOMIAL

A real number α is a zero of polynomial $f(x)$ if $f(\alpha) = 0$.

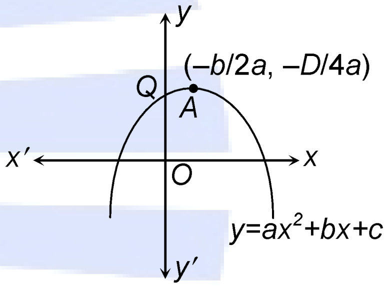
The zero of a linear polynomial $ax + b$ is $-\frac{b}{a}$. i.e. $-\frac{\text{Constant term}}{\text{Coefficient of } x}$

Geometrically zero of a polynomial is the point where the graph of the function cuts or touches *x*-axis.
When the graph of the polynomial does not meet the *x*-axis at all, the polynomial has no real zero.

1.4 SIGNS OF COEFFICIENTS OF A QUADRATIC POLYNOMIAL

The graphs of $y = ax^2 + bx + c$ are given in figure. Identify the signs of a, b and c in each of the following:

(i) We observe that $y = ax^2 + bx + c$ represents a parabola opening downwards. Therefore, $a < 0$.



We observe that the turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola is in first quadrant where $D = b^2 - 4ac$

$\therefore \quad -\frac{b}{2a} > 0 \Rightarrow -b < 0 \Rightarrow b > 0$

[∵ $a < 0$]

Parabola $y = ax^2 + bx + c$ cuts *y*-axis at *Q*.

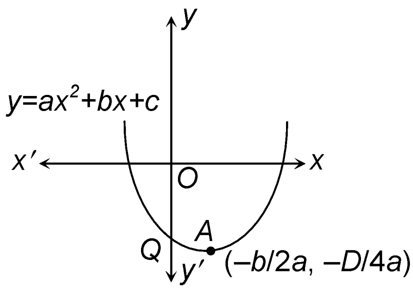
On *y*-axis, we have *x* = 0.

Putting x = 0 in $y = ax^2 + bx + c,$ we get $y = c.$

So, the coordinates of *Q* are (0, c). As *Q* lies on the positive direction of *y*-axis. Therefore, c > 0.

Hence, $a < 0, b > 0$ and $c > 0.$

(ii) We find that $y = ax^2 + bx + c$ represents a parabola opening upwards. Therefore, $a > 0$. The turning point of the parabola is in fourth quadrant.



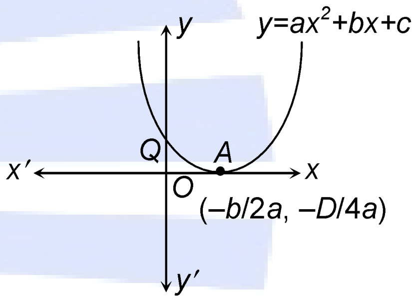
$\therefore \quad \frac{-b}{2a} > 0 \Rightarrow -b > 0 \Rightarrow b < 0.$

[$\because a > 0$]

Parabola $y = ax^2 + bx + c$ cuts y-axis at Q and y-axis. We have $x = 0$. Therefore, on putting $x = 0$ in $y = ax^2 + bx + c,$ we get $y = c.$

So, the coordinates of Q are $(0, c)$. As Q lies on negative y-axis. Therefore, $c < 0$.
Hence, $a > 0$, $b < 0$ and $c < 0$.

(iii) Clearly, $y = ax^2 + bx + c$ represents a parabola opening upwards.
Therefore, $a > 0$. The turning point of the parabola lies on OX.



$\therefore \quad -\frac{b}{2a} > 0 \Rightarrow -b > 0 \Rightarrow b < 0$

[$\because a > 0$]

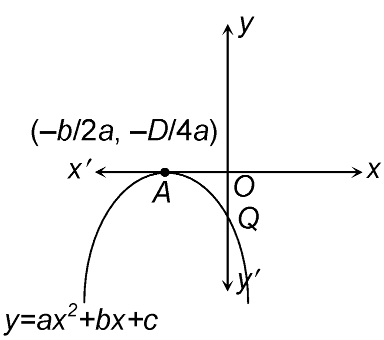
The parabola $y = ax^2 + bx + c$ cuts y-axis at Q which lies on positive y-axis. Putting $x = 0$ in $y = ax^2 + bx + c,$ we get $y = c$. So, the coordinates of Q are $(0, c)$. Clearly, Q lies on OY.

$\therefore \quad c > 0.$

Hence, $a > 0$, $b < 0$, and $c > 0$.

(iv) The parabola $y = ax^2 + bx + c$ opens downwards. Therefore, $a < 0$.

The turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola is on negative x-axis,



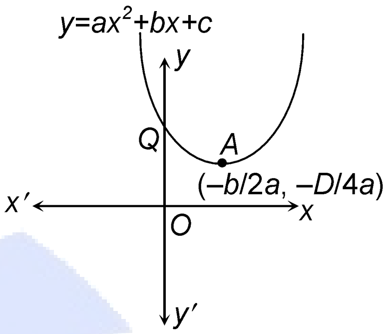
$\therefore \quad -\frac{b}{2a} < 0 \Rightarrow b < 0$ [$\because a < 0$]

Parabola $y = ax^2 + bx + c$ cuts *y*-axis at *Q* (0, *c*) which lies on negative y-axis. Therefore, $c < 0$.

Hence, $a < 0, b < 0$ and $c < 0$.

(v) We notice that the parabola $y = ax^2 + bx + c$ opens upwards. Therefore, $a > 0$.

Turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola lies in the first quadrant.



$\therefore \quad -\frac{b}{2a} > 0 \Rightarrow \frac{b}{2a} < 0 \Rightarrow b < 0$

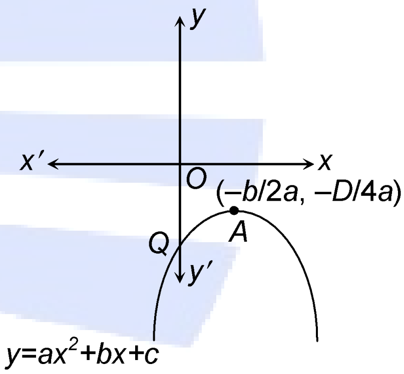
$[\because a > 0]$

As *Q* (0, c) lies on positive *y*-axis. Therefore, $c > 0$.

Hence, $a > 0, b < 0$ and $c > 0$.

(vi) Clearly, $a < 0$

$Q\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ lies in the fourth quadrant.



$\therefore \quad -\frac{b}{2a} > 0 \Rightarrow \frac{b}{2a} < 0 \Rightarrow b > 0 \qquad [\because a < 0]$

As *Q* (0, c) lies on negative *y*-axis.
Therefore, $c < 0$.

Hence, $a < 0, b > 0$ and $c < 0$.

5. GRAPH OF QUADRATIC POLYNOMIALS

In set theoretic language the graph of a polynomial $f(x)$ is the collection (or set) of all points (x, y), where $y = f(x)$. The graph of a polynomial $f(x)$ is a smooth free hand curve passing through every point of $f(x)$.

To draw a graph of polynomial $f(x)$, we may follow some steps.

1. Write the given quadratic polynomial $f(x) = ax^2 + bx + c$ as

$y = ax^2 + bx + c$

2. Calculate the zeros of the polynomial, if exist, by putting $y = 0$

   i.e., $ax^2 + bx + c = 0$

3. Calculate the points where the curve meets y-axis by putting $x = 0$.

4. Calculate $D = b^2 - 4ac$

   if $D > 0$, graph cuts x-axis at two points.
   $D = 0$, graph touches x-axis at one point.
   $D < 0$, graph is far away from x-axis.

5. Find $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ which is the turning point of curve.

6. Make a table of selecting values of x and corresponding values of y, two to three values on left and two to three values on right of turning point

7. Draw a smooth curve through these points by free hand. The graph so obtained is ` called a parabola.

3. RELATION BETWEEN THE ZEROES AND THE COEFFICIENTS OF A POLYNOMIAL

1. Quadratic polynomial: $ax^2 + bx + c = 0$; $a \neq 0$.

   Let α, β are two zeros of the given polynomial.

   Sum of zeros (α + β) = $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{b}{a}$

   Product of zeros (αβ) = $\frac{\text{constant}}{\text{coefficient of } x^2} = \frac{c}{a}$

2. Cubic polynomial: $ax^3 + bx^2 + cx + d = 0$; $a \neq 0$

   Let α, β and γ are three zeros of the given polynomial.

   (i) Sum of zeros $(\alpha + \beta + \gamma) = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3} = -\frac{b}{a}$

   (ii) Product of zeros taken two at a time

   $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{coefficient of } x}{\text{coefficient of } x^3} = \frac{c}{a}$

   (iii) Product of zeros (αβγ) = $-\frac{\text{constant}}{\text{coefficient of } x^3} = -\frac{d}{a}$

3. Formation of Quadratic Polynomial:

   Let α, β are the zeros, then required polynomial is

   $k[x^2 - (\text{sum of zeros})x + (\text{product of zeros})]$ or $k[(x-\alpha)(x-\beta)]$

   where k is a non-zero constant

4. Formation of Cubic Polynomial

Let α, β, γ are the zeros then required polynomial is

$$k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma] \quad \text{or} \quad k[(x-\alpha)(x-\beta)(x-\gamma)]$$

where k is a non-zero constant

4. DIVISION ALGORITHM FOR POLYNOMIALS

Let $p(x)$ and $g(x)$ be polynomials of degree n and m respectively such that $m \le n$. Then there exist unique polynomials $q(x)$ and $r(x)$ where $r(x)$ is either zero polynomial or degree of $r(x) <$ degree of $g(x)$ such that $p(x) = q(x) \cdot g(x) + r(x)$.

Dividend = Quotient × Divisor + Remainder.

$p(x)$ is dividend, $g(x)$ is divisor.

$q(x)$ is quotient, $r(x)$ is remainder.