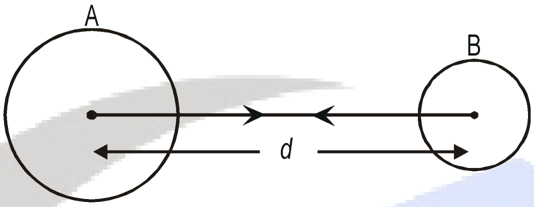
1. NEWTON'S UNIVERSAL LAW OF GRAVITATION

Newton gave the universal law that gave the relationship between the force of attraction between two bodies lying at certain distance. According to Newton's universal law of gravitation :

"Every object in the universe attracts every other object with a force. This force of attraction between any two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them".

The direction of force is along the line joining the centers of two objects.



Let two objects *A* and *B* of masses *M* and *m* lie at a distance d from each other as shown in figure. Let the force of attraction between two objects be *F*. According to the universal law of gravitation, the force between two objects is directly proportional to the product of their masses. That is,

$$F \propto M \times m \quad \ldots(i)$$

And the force between two objects is inversely proportional to the square of the distance between them, that is,

$$F \propto \frac{1}{d^2} \quad \ldots(ii)$$

Combining equation (i) and (ii)

$$F \propto \frac{M \times m}{d^2}$$

or

$$F = G\frac{M \times m}{d^2}$$

where G is the constant of proportionality and is called as universal gravitation constant.

$$F = G\frac{M \times m}{d^2}$$

$$F \times d^2 = GM \times m$$

$$G = \frac{F \times d^2}{M \times m} \quad \ldots(iii)$$

The SI unit of G can be obtained by substituting the unit of force, distance and mass in equation (iii).

$$G = \frac{Nm^2}{kg \times kg}$$

$G = Nm^2/kg^2$ or $G = Nm^2\,kg^{-2}$

The value of G was found out by Henry Cavendish (1731 – 1810) by using a sensitive balance.

The accepted value of G is $6.673 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$.

1.1 IMPORTANCE OF THE UNIVERSAL LAW OF GRAVITATION

The universal law of gravitation successfully explained several phenomena which were believed to be unconnected.

1. It is the gravitational force between the sun and the earth which keeps the earth in uniform circular motion around the sun.
2. The gravitational force between the earth and the moon makes the moon revolve at uniform speed around the earth. Thus the gravitational force is responsible for the existence of our solar system.
3. The tides in the sea formed by the rising and falling of water level in the sea, are due to the force of attraction which the sun and the moon exert on the water surface in the sea.
4. The gravitational force of the earth is responsible for holding the atmosphere above the earth.
5. It is also responsible for rain falling on the earth and for the flow of rivers.
6. It is also the gravitational force of the earth which keeps us firmly on the ground.

2. KEPLER'S LAWS OF PLANETARY MOTION

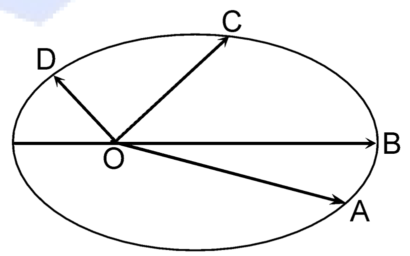
There has always been a great interest in the motion of planets. By the 16th century, a lot of data on the motion of planets had been collected by many astronomers. Based on these data Johannes Kepler derived three laws, which govern the motion of planets. These are called Kepler's laws. These are :

1. The orbit of a planet is an ellipse with the sun at one of the foci, as shown in the figure given below. In this figure O is the position of the sun.
2. The line joining the planet and the sun sweep equal areas in equal intervals of time. Thus if the time of travel from A to B is the same as that from C to D, then the areas OAB and OCD are equal.
3. The cube of the mean distance of a planet from the Sun is proportional to the square of its orbital period T. Or, r^3 / T^2 = constant.

It is important to note that although Kepler gave laws of planetary motion but could not give a theory to explain the motion of planets. It was Newton who showed that the cause of the planetary motion is the gravitational force that the Sun exerts on them.

2.1 HOW DID NEWTON GUESS THE INVERSE-SQUARE RULE?

Newton used the third law of kepler to calculate the gravitational force of attraction. The gravitational force of the earth is weakened by distance. A simple argument goes like this. We can assume that the planetary orbits are circular. Suppose the orbital velocity is v and the radius of the orbit is r. Then, the force acting on an orbiting planet is given by $F \propto v^2 / r$.



If T denotes the period, then $v = 2\pi r/T$, so that $v^2 \propto r^2/T^2$. (since 2π is constant)

We can rewrite this as $v^2 \propto (1/r) \times (r^3/T^2)$. Since r^3/T^2 is constant by Kepler's third law, we have $v^2 \propto 1/r$. Combining this with $F \propto v^2/r$, we get, $F \propto 1/r^2$. Thus, the gravitational force between the sun and a planet is inversely proportional to the square of distance between them.

3. FREE FALL

When we drop a body say a stone, we observe that its speed increases as it falls towards the earth. We have already seen that the body falls towards earth due to gravitational force between body and the earth. Whenever objects fall towards the earth under this force alone, we say that the objects are under free fall.

While falling, there is no change in the direction of motion of the objects. But due to the earth's attraction, there will be a change in the magnitude of the velocity. Any change in velocity involves acceleration.

The question is now whether we can measure its acceleration? Will this acceleration be more for heavier bodies?

If we drop stone and paper from a top of a house, we find that stone reaches the earth earlier than the paper. It was therefore thought that the acceleration for heavier bodies is more than lighter bodies.

Galileo decided to test this common belief. He dropped two stones of different masses from the top of leaning Tower of Pisa. It was found that both reached the earth in almost same time. He concluded that all the bodies fall towards the earth with equal acceleration. The acceleration with which the bodies fall towards the earth is independent of the masses of the bodies.

Reason for slowing down of lighter bodies was attributed to the resistance or friction offered by the air.

Robert Boyle took a vacuum pump to remove air from a tube containing a heavy coin and a sheet of paper. When the tube was inverted, both the coin and the paper hit the bottle at the same time. Thus, Galileo's prediction that all bodies fall with the same acceleration towards earth stands confirmed.

4. ACCELERATION DUE TO GRAVITY

When a body is dropped from a certain height, it falls with a constant acceleration.

This uniform acceleration produced in a freely falling body due to the gravitational pull of the earth is known as acceleration due to gravity and it is denoted by the letter '*g*'.

Although *g* varies very slightly from place to place but its average value is taken to be 9.8 m/s^2.

This means that the velocity of a body increases by 9.8 m/s every second. Say, if the body is dropped with zero velocity, its velocity becomes 9.8 m/s after 1s; 19.6 m/s after 2s; 27.4 m/s after 3s and so on. Similarly, if a body is projected upwards, its velocity decreases by 9.8 m/s after every second.

4.1 CALCULATION OF VALUE OF '*g*'

Case I: If we drop a body (say, a stone) of mass '*m*' from a distance 'd' from the center of the earth of mass *M*, then the force exerted by the earth on the stone is given by Newton's law of gravitation as:

$$F = G \times \frac{M \times m}{d^2} \qquad \text{… (i)}$$

We also know from the second law of motion that force is the product of mass and acceleration. We already know that there is acceleration involved in falling objects due to the gravitational force and is denoted by *g*.

Therefore, the magnitude of the gravitational force F will be equal to the product of mass and acceleration due to the gravitational force, that is,

$$F = m \times g \quad \ldots\text{(ii)}$$

From equation (i) and (ii)

$$m \times g = G\frac{M \times m}{d^2}$$

$$g = G\frac{M}{d^2}$$

Case-II

Let an object be on or near the surface of the earth. The distance d will be equal to R, the radius of the earth. Thus, for objects on or near the surface of the earth,

$$mg = G\frac{M \times m}{R^2}$$

$$g = G\frac{M \times m}{R^2}$$

The earth is not a perfect sphere. As the radius of the earth increases from the poles to the equator, the value of 'g' becomes greater at the poles than at the equator.

To calculator the value of g, we should put the values of $G = 6.7 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$; mass of the earth $M = 6 \times 10^{24}\,\text{kg}$ and radius of the earth (R) $= 6.4 \times 10^6\,\text{m}$

$$g = G\frac{M}{R^2}$$

Value of *g* on earth

$$= \frac{6.7 \times 10^{-11} Nm^2\,/\text{kg}^2 \times 6 \times 10^{24}\,\text{kg}}{(6.4 \times 10^6\,\text{m})^2}$$

$$= 9.8\,\text{m/s}^2$$

Thus, value of acceleration due to gravity of the earth, $g = 9.8\,\text{m/s}^2$.

Value of *g* on moon

Mass of moon $= 7.4 \times 10^{22}\,\text{kg}$

Radius of moon $= 1{,}740\,\text{km}$

$$= 1{,}740{,}000\,\text{m} = 1.74 \times 10^6\,\text{m}$$

$$g = G\frac{M}{R^2} = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(1.74 \times 10^6)^2} = 1.63\ \text{m/s}^2$$

5. EQUATIONS OF MOTION OF FREELY FALLING BODIES

When the bodies are falling under influence of gravity, they experience acceleration *g* i.e. 9.8 m/s^2.

However, when these are going up against gravity they move with retardation of 9.8 m/s^2.

All the gravitation of motion already read by us are valid for freely falling bodies with the difference that a is replaced by *g*. For motion vertically upward a is replaced by *g*. Here '*s*' is also replaced by *h*, the height.

$v = u + at$	changes to	$v = u + gt$
$s = ut + \frac{1}{2}at^2$	changes to	$h = ut + \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$	changes to	$v^2 = u^2 + 2gh$

6. MASS AND WEIGHT

The similarities and differences between mass and weight are discussed as follows :

6.1 MASS

The mass of a body is the quantity of matter contained in it.
Mass is a scalar quantity. The unit of mass is kilogram.
A body contains the same quantity of matter whether it be on the earth, moon or even in outer space. Thus, the mass of a body is constant and does not change from place to place.
Mass of a body is usually denoted by the small '*m*'.
Mass of a body is a measure of inertia of the body and hence it is also known as inertial mass.

6.2 WEIGHT

We know that the earth attracts every object with a certain force and this force depends on the mass (m) of the object and the acceleration due to gravity (g).
The weight of an object is the force with which it is attracted towards the earth.

We know that, $F = m \times a$

That is, $F = m \times g$

The force of attraction of the earth on an object is known as the weight of the object. It is denoted by W.

So we have, $W = m \times g$

As the weight of an object is the force with which it is attracted towards the earth, the S.I. unit of weight is the same as that of force i.e. Newton (N).
The weight is a force acting vertically downwards; it has both magnitude and direction, so it is a vector quantity.
The value of g is constant at a given place. Therefore at a given place, the weight of an object is directly proportional to the mass, say m, of the object, that is, $W \propto m$. It is due to this reason that at a given place, we can use the weight of an object as a measure of its mass.

The mass of an object remains the same everywhere, that is, on the earth and on any planet whereas its weight depends on its location.

6.1.1 Weight of a freely falling body

Let us suppose that a body is placed on a lift, the weighing machine will show the weight of the body on its scale. Now, the lift is made to fall freely due to gravity, both the weighing machine as well as the body will fall with same acceleration i.e., with g in the downward direction. The body will, therefore, not press the weighing machine with any force and hence show zero weight. Thus a body is weightless during free fall.

6.1.2 Weightlessness in space

Consider an astronaut in a space ship orbiting the earth about 1000 km above its surface. At that distance from the earth, the force of gravity of earth is still quite strong. Since the acceleration due to gravity is not zero, the weight of astronaut in the space ship certainly cannot be zero. But we all have seen them on T.V., floating is a space ship and believe that in this situation they are weightless. This can be explained as follows:

When an astronaut in the space ship is orbiting the earth, then both, the astronaut and the spaceship are in a continuing state of free fall towards the earth with the same acceleration due to gravity. Since the downward acceleration of the astronaut is the same as that of the spaceship he does not exert any force on the sides of the space ship and a weighing machine kept in the space vehicle will show his weight to be zero. Though the free fall of a body produces a feeling of weightlessness but a true weightlessness can be experienced by a spaceship in a region of outer space where the acceleration due to gravity '*g*' is zero.

6.1.3 Weight of an object on the moon

We have learnt that the weight of an object on the earth is the force with which the earth attracts the object.

In the same way, the weight of an object on the moon is the force with which the moon attracts that object.

The mass of the moon is less than that of the earth. Due to this, the moon exerts lesser force of attraction on objects.

Let the mass of an object be m. Let its weight on the moon be W_m. Let the mass of the moon be N_m and its radius be R_m.

By applying the universal law of gravitation, the weight of the object on the moon will be

$$W_m = G\frac{M_m \times m}{R_m^2}$$

If the weight of the same object on the earth be W_e. The mass of the earth is *M* and its radius is *R*.

Now, $$W_e = G\frac{M \times m}{R^2}$$

Mass of earth $= 5.98 \times 10^{24}$ kg

Radius of earth $= 6.37 \times 10^{6}$ m

Mass of moon $= 7.36 \times 10^{22}$ kg

Radius of moon $= 1.74 \times 10^{6}$ m

Substituting the values:

$$W_m = G\frac{7.36\times10^{22}\,kg\times m}{(1.74\times10^6\,m)^2}$$

$$W_m = 2.431\times10^{10}\,G\times m \qquad \ldots\text{(i)}$$

And $$W_e = 1.474\times10^{11}\,G\times m \qquad \ldots\text{(ii)}$$

Dividing equation (i) by (ii), we get

$$\frac{W_m}{W_e} = \frac{2.431\times10^{10}}{1.474\times10^{11}}$$

or $$\frac{W_m}{W_e} = 0.165 \approx \frac{1}{6}$$

$$\frac{\text{Weight of the object on the moon}}{\text{Weight of the object on the earth}} = \frac{1}{6}$$

∴ Weight of the object on the moon = (1/6) × weight of the object on the earth.

7. THRUST AND PRESSURE

The force acting on an object perpendicular to the surface is called thrust. Let us understand the meaning of thrust and pressure practically.

Situation-1:

You fix a poster on a notice board and while doing so you need to press drawing pins with your thumb. So pressing drawing pins means applying force on the surface area of the head of the pin. This force is directed perpendicular to the surface area of the board.

Situation-2:

When you stand on loose sand your feet go deep into the sand. But when you lie down on the sand, you will find that your body will not go deep on the sand.

This is because when you stand on loose sand, the force i.e., the weight of your body is acting on an area equal to area of your feet. When you lie down, the same force acts on an area equal to the contact area of your whole body, which is larger than the area of your feet.

Thus, the effect of thrust on sand is larger while standing than while lying.

The thrust on unit area is called pressure. Thus,

$$\text{Pressure} = \frac{\text{thrust}}{\text{area}}.$$

S.I. unit of pressure is N/m^2 or Nm^{-2}.

In honour of scientist Blaise Pascal, the S.I. unit of pressure is called pascal, denoted as Pa.

Pressure depends on two factors :

(i) Force applied

(ii) Area over which force acts.

8. PRESSURE AND FLUIDS

All liquids and gases are called as fluids. A solid exerts pressure on a surface due to its weight. Similarly, fluids have weight, and they also exert pressure on the base and walls of the container in which they are enclosed. A fluid exerts pressure in all directions even upwards.

8.1 BUOYANCY

To understand the term buoyancy, let us perform an activity :

Take an empty plastic bottle. Close the mouth of the bottle with an air tight stopper. Put it in a bucket filled with water.

- We will see that the bottle floats.
- Now push the bottle into the water. We will feel an upward push. If we push it further down, we will find it difficult to push deeper and deeper.
- This indicates that water exerts a force on the bottle in the upward direction.
- This upward force exerted by the water goes on increasing as the bottle is pushed deeper till it is completely immersed.
- If we release the bottle, it bounces back to the surface.

Explanation

The force due to the gravitational attraction of the earth acts on the bottle in the downward direction. So the bottle is pulled downwards. But the water exerts an upward force on the bottle. Thus, the bottle is pushed upwards.

The weight of an object is the force due to gravitational attraction of the earth. When the bottle is immersed, the upward force exerted by the water on the bottle is greater than its weight therefore it rises up, when released.

To keep the bottle completely immersed the upward force on the bottle due to water must be balanced. This can be achieved by an externally applied force acting downwards. This force must at least be equal to the difference of upward force and the weight of the bottle.

The upward force acting on an object immersed in a liquid is called buoyant force or upthrust and this phenomenon of exerting beyond force is called buoyancy.

8.2 FACTORS AFFECTING BUOYANT FORCE

The magnitude of buoyant force acting on an object immersed in a liquid depends on two factors :

(i) Volume of the object immersed in the liquid.

The buoyant force exerted by a liquid depends on the volume of the solid object immersed in the liquid. As the volume of solid object immersed inside the liquid increases, the upward buoyant force also increases. And when the object is completely immersed in the liquid, the buoyant force becomes the maximum and remain constant. Also, the magnitude of buoyant force acting on a solid object does not depend on the nature of the solid object.

It depends only on its volume. For e.g. If two balls made of different metals having different weights but equal volumes are fully immersed is a liquid, they will experience an equal upward ‘buoyant force’ i.e. equal loss in weight.

(ii) Density of the liquid.

The buoyant force exerted by a liquid depends on the density of the liquid in which the object is immersed. As the density of liquid increases, the buoyant force exerted by it also increases, for example sea water has higher density then fresh water therefore sea water will exert more buoyant force on an object immersed in it than the fresh water. Therefore it is easier to swim in sea water because sea water exerts a greater buoyant force on the swimmer due to its higher density.

8.3 ARCHIMEDES' PRINCIPLE

Archimedes was a Greek scientist. He discovered the principle, subsequently named after him, after noticing that the water in a bath tub overflowed when he stepped into it. He ran through the streets shouting "Eureka"! which means "I have got it". This knowledge helped him to determine the purity of the gold in the crown made for the king. This work in the field of geometry and mechanics made him famous. His understanding of levers, pulleys, wheels–axle helped the Greek army in its war with roman army.

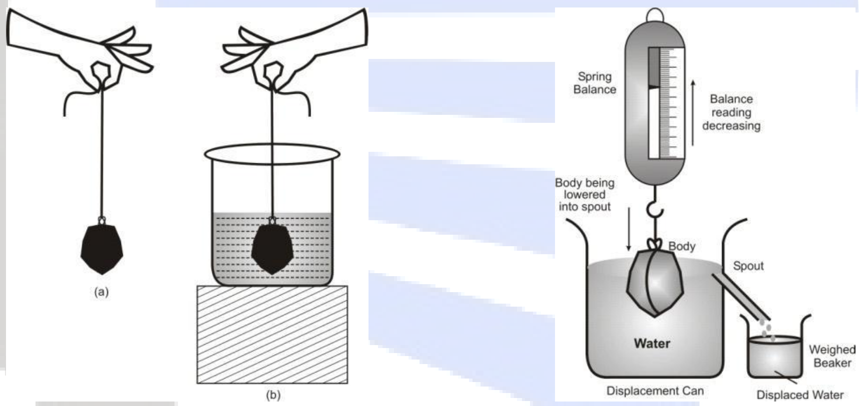
"Archimedes' principle states that when a body is partially or wholly immersed in a liquid, it experiences an upward force that is equal to the weight of the fluid displaced by it.

8.3.1 Activity to understand Archimedes' principle

(a) Take a piece of stone and tie it to one end of a rubber string or a spring balance.
(b) Suspend the stone by holding the balance or the string as shown in figure below.
(c) Note the elongation of the string or the reading on the spring balance due to the weight of the stone.
(d) Now, slowly dip the stone in the water in a container. You will find that the elongation of the string or the reading of the balance decreases as the stone is gradually lowered in the water.
(e) However no further change is observed once the stone gets fully immersed in the water.

Explanation:

The elongation is produced in the string or the spring balance due to the weight of the stone. Since the extension decreases once the stone is lowered in water, it means that same force acts on the stone in upward direction. As a result, the net force on the string decreases and hence the elongation also decreases. This upward force exerted by water is known as the force of buoyancy.



8.3.2 Applications of Archimedes' principle

(a) Archimedes' principle is used in determining the relative density of a substance.
(b) The hydrometers used for determining the density of liquids are based on Archimedes' principle.
(c) The lactometers used for determining the purity of milk are based on Archimedes' principle.
(d) Archimedes' principle is used in designing ships and submarines.

8.3.3 Why objects float or sink in a liquid?

When an object is put in a liquid, then two forces act on it :

- Weight of an object acting downwards due to the gravitational pull of the earth on the object.
- Buoyant force acting upwards which tends to push the object up.

(a) Sinking of an object in water

If we place an iron nail on the surface of water in a beaker then the nail sinks. The force due to the gravitational attraction of the earth on the iron nail pulls it downwards. There is an upthrust of water on the nail, which pushes it upwards. But the downward force acting on the nail is greater than the upthrust of water on the nail, so it sinks.

(b) Floating of an object in water

If we place a piece of cork on the surface of water in a beaker, then the cork floats. This happens because of the difference in their densities. The density of a cork is less than the density of water. This means that the upthrust of water on the cork is greater than the weight of the cork. So, it floats.

Therefore, objects of density less than that of a liquid float on the liquid. The objects of density greater than that of a liquid sink in the liquid.

9. DENSITY

We describe the lightness or heaviness of different substances by using the word density.

The density of a substance is defined as mass of the substance per unit volume. That is,

$$\text{Density} = \frac{\text{mass of the substance}}{\text{vol. of the substance}}$$

SI unit is kg/m^3 or $kg\,m^{-3}$

The density of a given substance, under specified conditions remains the same. Therefore, the density of a substance is one of the characteristic property of a substance.

For example, density of gold is $19300\,kg/m^3$, while that of water is $1000\,kg/m^3$.

The density of a given sample of a substance can help us to determine its purity.

10. RELATIVE DENSITY

The relative density of a substance is the ratio of its density to that of water.

$$\text{Relative density of a substance} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

Since, the relative density is a ratio of similar quantities, it has no unit.

The relative density of a substance expresses the heaviness of the substance in comparison to water. For example, the relative density of iron is 7.8. This means that iron is 7.8 times as heavy as an equal volume of water.

The relative density of water is 1. Now if the relative density of a substance is more than 1, then it will be heavier than water and hence it will sink in water. On the other hand, if the relative density of a substance is less than 1, it will be lighter than water and hence will float in water.