

## INTRODUCTION TO EUCLID'S GEOMETRY

### EUCLID'S DEFINITIONS

**Points** : A point is that which has no part.  
**Line** : A line is breadthless length.  
**Surface** : A surface is that which has length and breadth only.  
**Ends of a line** : The ends of a line are points.  
**Edges of a Surface** : The edges of a surface are lines.  
**Straight line** : A straight line is a line which lies evenly with the points on itself.  
**Plane Surface** : A plane surface is a surface which lies evenly with the straight lines on itself.

### EUCLID'S POSTULATES

Postulate 1 : A straight line may be drawn from any one point to any other point.  
Postulate 2 : A terminated line (i.e., a line segment) can be produced indefinitely on either side.  
Postulate 3 : A circle can be drawn with any centre and any radius.  
Postulate 4 : All right angles are equal to one another.  
Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

### EUCLID'S AXIOMS

1. Things which are equal to the same thing are equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equal are subtracted from equals, the remainders are equal.
4. The things which coincide with one another are equal.
5. The whole is greater than the part.
6. Things which are double the same things are equal to one another.
7. Things which are halves of the same things are equal to one another.

### EQUIVALENT VERSIONS OF EUCLID'S FIFTH POSTULATE

There are several equivalent versions of the fifth postulate of Euclid. One such version is stated as 'Playfair's Axiom' which is given below :

#### Playfair's Axiom (Axiom for Parallel Lines)

For every line  $\ell$  and for every point P not lying on  $\ell$ , there exists a unique line m passing through P and parallel to  $\ell$ .

Another version of the above axiom is as stated below :

Two distinct intersecting lines cannot be parallel to the same line.